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Lecture 1

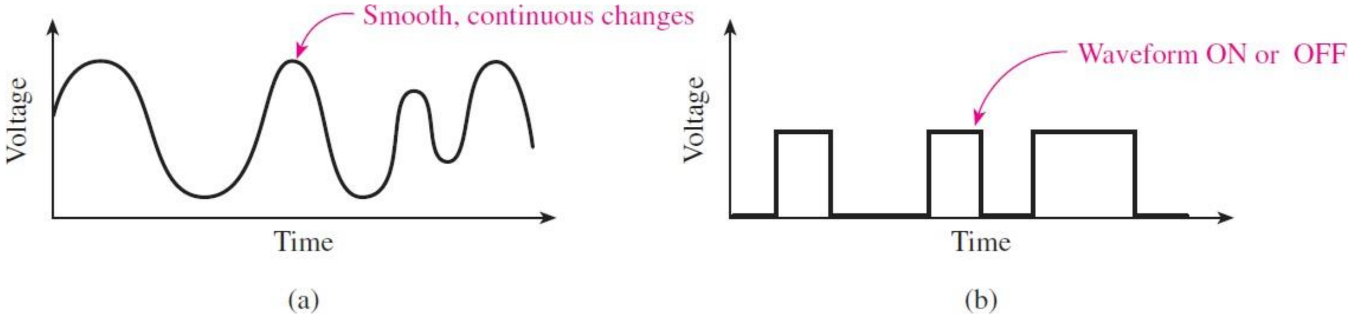
Contents

1. Introduction to digital electronics 3
2. Numbering systems 4
2.1. Decimal Numbering System (Base 10) 4
2.2. Binary Numbering System (Base 2) 5
2.2.1. Binary to Decimal conversion 5
2.2.2. Decimal to Binary conversion 6
2.3 Octal Numbering System (Base 8) 7
2.3.1. Binary–Octal and Octal–Binary Conversions 9
2.4. Hexadecimal Numbering System (Base 16) 10
2.5. Binary-Coded-Decimal System (BCD) 12
3. Arithmetic Operations 13
3.1. Addition 13
3.2. Subtraction 17
3.2.1. Binary Subtraction 17
3.2.1. Octal Subtraction 18
3.2.1. Hexa Subtraction 18
3.3. Multiplication 19
3.1. Division 21
4. Binary system complements 21
4.1. 1's complement 22
4.2. 2's complement 22
5. Exercises 23

1. Introduction to digital electronics

Digital Electronics is the sub-branch of electronics which deals with digital signals for processing and controlling various systems and sub-systems. In various applications like sensors and actuators, usage of digital electronics is increasing extensively.

Digital electronics is entirely the field in which digital signals is used. Digital signals are discretization of analog signals. A signal carries information. In digital signals the values in a particular band is same i.e. constant. Digital signals form the basis of digital circuit and digital electronics.



Digital signals can be represented with two numbers or states, in most cases, the number of these states is two, and they are represented by two voltage bands: one near a reference value (typically termed as "ground" or zero volts), and the other a value near the supply voltage. These correspond to the "false" ("0") and "true" ("1") values of the Boolean domain respectively, named after its inventor, George Boole, yielding binary code.

Digital techniques are useful because it is easier to get an electronic device to switch into one of a number of known states than to accurately reproduce a continuous range of values. Digital electronic circuits are usually made from large assemblies of logic gates, simple electronic representations of Boolean logic functions. A digital circuit is typically constructed from small electronic circuits called logic gates that can be used to create combinational logic. Each logic gate is designed to perform a function of Boolean logic when acting on logic signals. A logic gate is generally created from one or more electrically controlled switches, usually transistors but thermionic valves have seen historic use. The output of a logic gate can, in turn, control or feed into more logic gates. Integrated circuits consist of multiple transistors on one silicon chip, and are the least expensive way to make large number of interconnected logic gates. Integrated circuits are usually designed by engineers using electronic design automation software (see below for more information) to perform some type of function.

2. Numbering systems

2.1. Decimal Numbering System (Base 10)

In the decimal numbering system, each position contains 10 different possible digits. These digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each position in a multidigit number will have a weighting factor based on a power of 10.

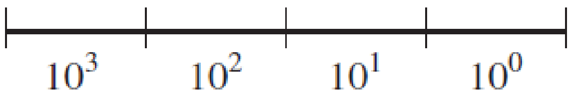


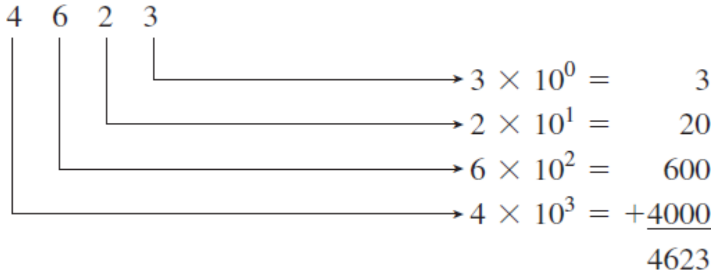
Figure 1 Weighting factors of four-digit decimal number.

The least significant position has a weighting factor of 10^0

The most significant position (leftmost) has aweighting factor of 10^3:

Example: To evaluate the decimal number 4623, the digit in each position is multiplied by the appropriate weighting factor:

Answer:



2.2. Binary Numbering System (Base 2)

Digital electronics use the **binary**numbering system because it uses only the digits 0 and 1, which can be represented simply in a digital system by two distinct voltage levels, such as +5 V = 1 and 0 V = 0.

2.2.1. Binary to Decimal conversion

The weighting factors for binary positions are the powers of 2 shown in Table 1–1.

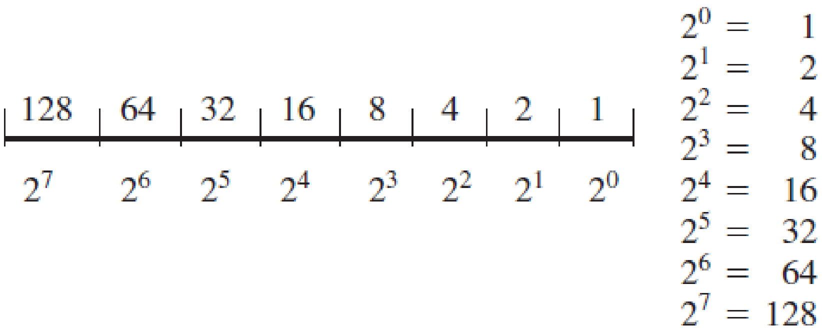
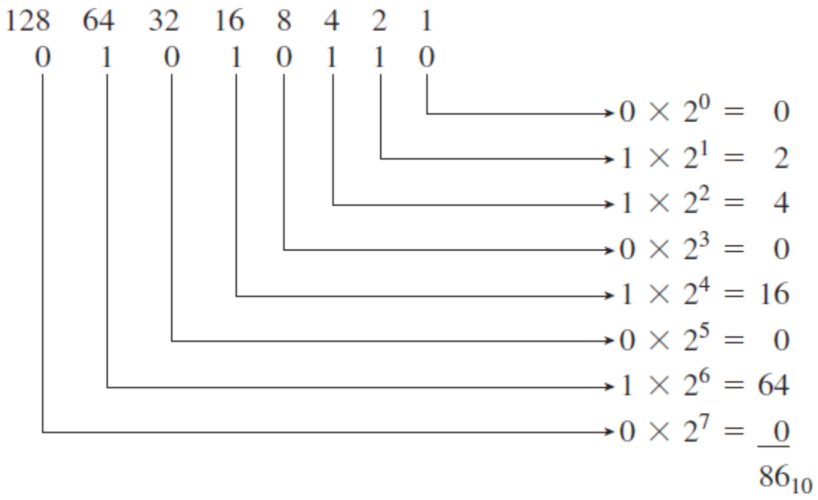


Figure 2 Weighting factors of 8-digit binary number.

Example: Convert the binary number **(01010110)**$_2$ to decimal?

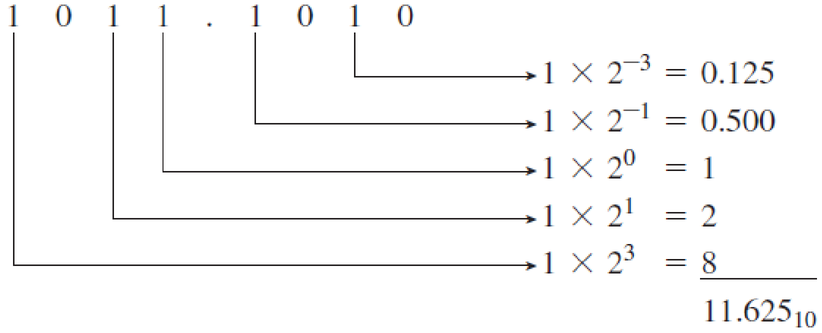
Answer:



$(01010110)_2 = (86)_{10}$

Example: Convert the fractional binary number **(1011.1010)₂** to decimal?

Answer:



$(1011.1010)_2 = (11.625)_{10}$

2.2.2. Decimal to Binary conversion

The binary equivalent can be found by successively dividing the integer part of the number by 2and recording the remainders until the quotient becomes ‘0’. The remainders written in reverse orderconstitute the binary equivalent.

Example: Convert **$(152)_{10}$** to binary number?

Answer:

$152 \div 2 = 76$	remainder 0	(LSB)
$76 \div 2 = 38$	remainder 0	
$38 \div 2 = 19$	remainder 0	
$19 \div 2 = 9$	remainder 1	
$9 \div 2 = 4$	remainder 1	
$4 \div 2 = 2$	remainder 0	
$2 \div 2 = 1$	remainder 0	
$1 \div 2 = 0$	remainder 1	(MSB)

$(152)_{10} = (10011000)_2$

<u>Example:</u> Convert**$(87)_{10}$**to binary number?

<u>Answer:</u>

2	**87**	**1**
2	**43**	**1**
2	**21**	**1**
2	**10**	**0**
2	**5**	**1**
2	**2**	**0**
2	**1**	**1**

$(87)_{10} = (1010111)_2$

<u>Example:</u> Convert**$(20.625)_{10}$**to binary number?

<u>Answer:</u>

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1

$0.625 \times 2 = 1.25$	1
$0.25 \times 2 = 0.5$	0
$0.5 \times 2 = 1$	1

$(20.625)_{10} = (10100.101)_2$

2.3. <u>Octal Numbering System (Base 8)</u>

The **octal**numbering system is a method of grouping binary numbers in groups of three. The eight allowable digits are 0, 1, 2, 3, 4, 5, 6, and 7. The octal numbering system is used by manufacturers of computers that utilize 3-bit codes to indicate instructions or operations to be performed. By using the octal representation instead of binary, the user can simplify the task of entering or reading computer instructions and thus save time.

Decimal	Binary	Octal
0	000	0
1	001	1
2	010	2
3	011	3
4	100	4
5	101	5
6	110	6
7	111	7
8	1000	10
9	1001	11
10	1010	12

<u>Example:</u> Convert **$(137.21)_8$** to decimal number?

<u>Answer:</u>

1 3 7 . 2 1

$1 \times 8^{-2} = 0.01562$

$2 \times 8^{-1} = 0.25$

$7 \times 8^{0} = 7$

$3 \times 8^{1} = 24$

$1 \times 8^{2} = 64$

95.26562

$(137.21)_8 = (95.26562)_{10}$

Example: Convert **$(73.75)_{10}$** to octal number?

Answer:

8	73	1
8	9	1
8	1	1

$0.75 \times 8 = 6$	6

$(73.75)_{10} = (111.6)_8$

2.3.1. Binary–Octal and Octal–Binary Conversions

An octal number can be converted into its binary equivalent by replacing each octal digit with its three-bit binary equivalent. We take the three-bit equivalent because the base of the octal number system is 8 and it is the third power of the base of the binary number system, i.e. 2. All we have then to remember is the three-bit binary equivalents of the basic digits of the octal number system. A binary number can be converted into an equivalent octal number by splitting the integer and fractional parts into groups of three bits, starting from the binary point on both sides. The 0s can be added to complete the outside groups if needed.

Example: Convert **$(624)_8$** to binary number?

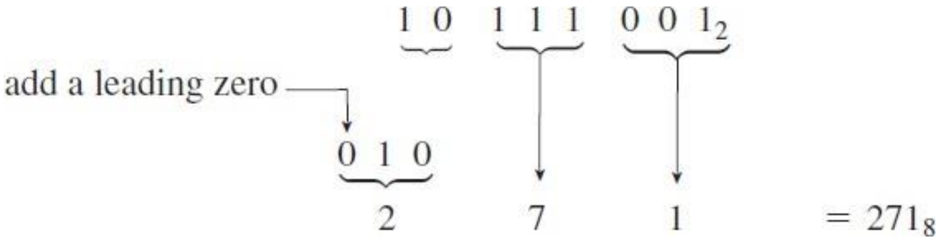
Answer:

$$\overbrace{1\;1\;0}^{6}\quad\overbrace{0\;1\;0}^{2}\quad\overbrace{1\;0\;0}^{4} = 1\;1\;0\;0\;1\;0\;1\;0\;0_2$$

$(624)_8 = (110010100)_2$

Example: Convert **$(10111001)_2$** to octal number?

Answer:



$(10111001)_2 = (271)_8$

2.4. Hexadecimal Numbering System (Base 16)

The hexadecimal numbering system, like the octal system, is a method of groupingbits to simplify entering and reading the instructions or data present in digital computersystems. Hexadecimal (hex) uses 16 different digits and is a method of grouping binary numbers in groups of four.

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F
16	0001 0000	1 0
17	0001 0001	1 1
18	0001 0010	1 2
19	0001 0011	1 3
20	0001 0100	1 4

The 16 allowable hex digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.

<u>Example:</u> Convert**$(2A6)_{16}$**to decimal number?

<u>Answer:</u>

2 A 6

$$6 \times 16^0 = 6 \times 1 = 6$$
$$A \times 16^1 = 10 \times 16 = 160$$
$$2 \times 16^2 = 2 \times 256 = \underline{512}$$
$$678_{10}$$

$(2A6)_{16} = (678)_{10}$

<u>Example:</u> Convert**$(82.25)_{10}$**to hexadecimal number?

<u>Answer:</u>

16	82	2 ↑
16	5	5

$0.25 \times 16 = 4$	4 ↓

$(82.25)_{10} = (52.4)_{16}$

To convert from binary to hexadecimal, group the binary number in groups of four(starting in the least significant position) and write down the equivalent hex digit.

<u>Example:</u> Convert **$(01101101)_2$** to hexadecimal number?

<u>Answer:</u>

$$\underbrace{0\ 1\ 1\ 0}_{6}\quad \underbrace{1\ 1\ 0\ 1_2}_{D} = 6D_{16}$$

$(01101101)_2 = (6D)_{16}$

<u>Example:</u> Convert **$(A9)_{16}$** to binary number?

<u>Answer:</u>

$$\overbrace{1\ 0\ 1\ 0}^{A}\quad \overbrace{1\ 0\ 0\ 1}^{9} = 10101001_2$$

$(A9)_{16} = (10101001)_2$

2.5. <u>Binary-Coded-Decimal System (BCD)</u>

The binary-coded-decimal (BCD) system is used to represent each of the 10 decimal digits as a 4-bit binary code. This code is useful for outputting to displays that are always numeric (0 to 9), such as those found in digital clocks or digital voltmeters.

To form a BCD number, simply convert each decimal digit to its 4-bit binary code.

Example: Convert**(496)$_{10}$**to BCD?

Answer:

$$\overbrace{0100}^{4}\ \overbrace{1001}^{9}\ \overbrace{0110}^{6} = 0100\ \ 1001\ \ 0110_{BCD}$$

(496)$_{10}$= (0100 1001 0110)$_{BCD}$

Example: Convert**(0111 0101 1000)$_{BCD}$**to decimal?

Answer:

$$\underbrace{0111}_{7}\ \underbrace{0101}_{5}\ \underbrace{1000}_{8} = 758_{10}$$

(0111 0101 1000)$_{BCD}$= (758)$_{10}$

Example: Convert**(0110 0100 1011)$_{BCD}$**to decimal?

Answer:

0110	0100	1011
6	4	*

This conversion is impossible because 1011 is not a valid binary-coded decimal. It is not in the range 0 to 9.

3. Arithmetic Operations

3.1. Addition

The following tables illustrate the rules of addition in Binary, Octal and hexadecimal.

Case	A + B	Sum	Carry
1	0 + 0	0	0
2	0 + 1	1	0
3	1 + 0	1	0
4	1 + 1	0	1

Figure 3 Binary addition rules.

A

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Sum

B

Figure 4 Octal addition rules.

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

X

Sum

Y

Figure 5 Hexadecimal addition rules.

Example: Add **(0011010)$_2$** to **(0001100)$_2$**?

Answer:

1 1 ← **Carry**

0 0 1 1 0 1 0

\+ **0 0 0 1 1 0 0**

0 100 1 1 0

Example: Find the result of **(11010.1101)$_2$+(111101.111)$_2$**?

Answer:

1 1 1 1 1 1 1   1 ← **Carry**

0 11010.1101

\+ **111101.111 0**

1 0 1 1 0 0 0 . 1 0 1 1

Example: Add $(7342)_8$ to $(11255)_8$?

Answer:

```
      1  1        ← Carry
      0 7342
  +
      11255
    ---------
    20617
```

Example: Add $(236.40)_8$ to $(542.76)_8$?

Answer:

```
    1 1 1 1       ← Carry
      2 3 6 . 4 0
  +
      5 4 2 . 7 6
    -------------
    1 0 0 1 . 3 6
```

Example: Find the results of $(44E1D)_{16}+(118B9)_{16}$?

Answer:

```
        1  1          ← Carry
      44E1D
  +
      118B9
    ----------------
      5 6 6 D 6
```

Example: Add $(44E.1D)_{16}$ to $(11F.6)_{16}$?

Answer:

```
      1          ← Carry
     44E.1D
  +
     11F.6
   ---------
     5 6 D . 7 D
```

3.2. Subtraction

3.2.1. Binary Subtraction

Subtraction and Borrow, these two words will be used very frequently for the binary subtraction. There are four rules of binary subtraction.

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Figure 6 Binary subtraction rules

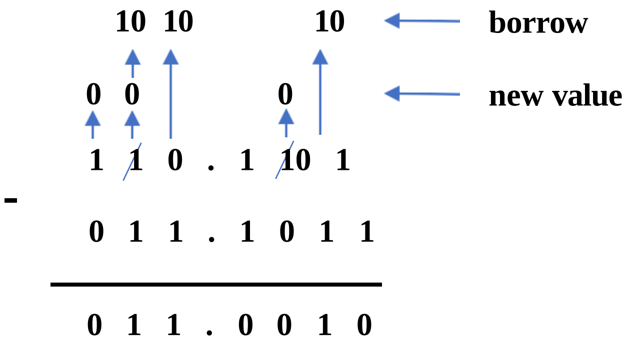
Example: Find the results of $(0011010)_2 - (001100)_2$?

Answer:

```
          10 10     ← borrow
         0  0       ← new value
     0011010
  -
       0 0 1 1 0 0
   ---------------
     0 0 0 1 1 1 0
```

Example: Find the results of **$(110.1101)_2$-$(11.1011)_2$**?

Answer:

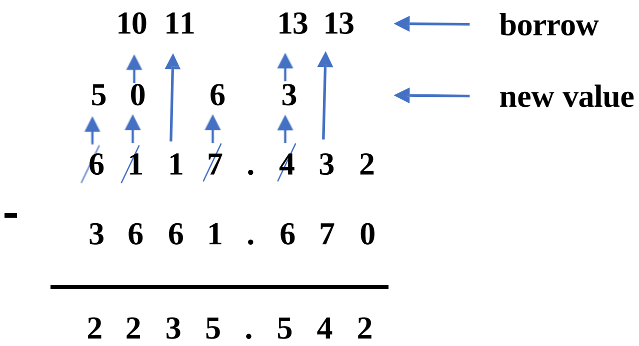


3.2.1. Octal Subtraction

The subtraction of octal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed number. In the decimal system, you borrow a group of 10_{10}. In the binary system, you borrow a group of 2_{10}. In the octal system you borrow a group of 8_{10}.

Example: Find the results of **$(6117.432)_8$- $(3661.67)_8$**?

Answer:

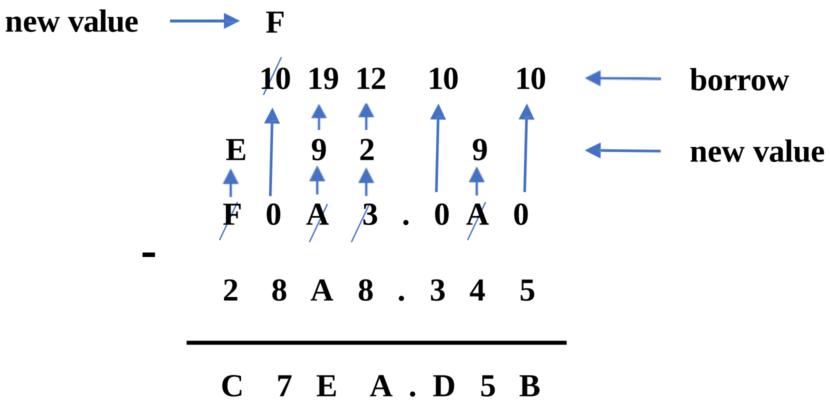


3.2.1. Hexa Subtraction

The subtraction of hexadecimal numbers follows the same rules as the subtraction of numbers in any other number system. The only variation is in borrowed number. In the decimal system, you borrow a group of 10_{10}. In the binary system, you borrow a group of 2_{10}. In the hexadecimal system you borrow a group of 16_{10}.

<u>Example:</u> Find the results of **$(F0A3.0A)_{16}$- $(28A8.345)_{16}$**?

<u>Answer:</u>



3.3. <u>Multiplication</u>

In this section we will discuss the multiplication rules only in binary numbers.

Binary multiplication is similar to decimal multiplication. It is simpler than decimal multiplication because only 0s and 1s are involved. There are four rules of binary multiplication.

Case	A x B	Multiplication
1	0 x 0	0
2	0 x 1	0
3	1 x 0	0
4	1 x 1	1

Figure 7 Binary multiplication rules

Example: Find the results of $\mathbf{(10011)_2 \times (0101)_2}$?

Answer:

1 0 0 1 1

0 1 0 1 ×

1 0 0 1 1

0 0 0 0 0

1 0 0 1 1

0 0 0 0 0

010 1 1111

Example: Find the results of $\mathbf{(100.111)_2 \times (010.11)_2}$?

Answer:

1 0 0 . 1 1 1

0 1 0 . 1 1

1 0 01 1 1

1 0 0 1 1 1

0 0 0 0 0 0

1 0 0 1 1 1

0 0 0 0 0 0

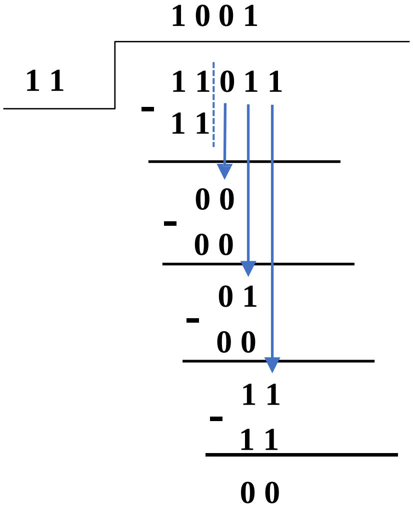
01101 . 01101

3.1. <u>Division</u>

Binary division is similar to decimal division. It is called as the long division procedure.

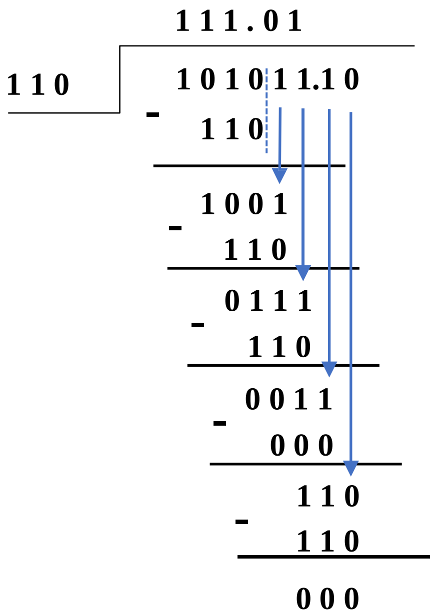
<u>Example:</u>Find the results of $\mathbf{(11011)_2 \div (11)_2}$?

<u>Answer:</u>



<u>Example:</u>Find the results of $\mathbf{(101011.10)_2 \div (110)_2}$?

<u>Answer:</u>



4. <u>Binary system complements</u>

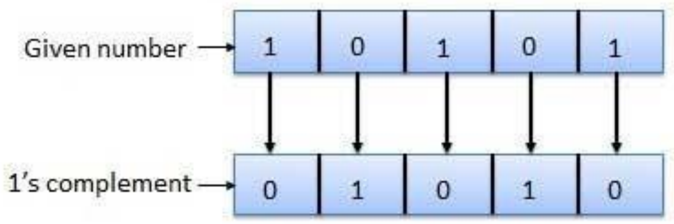
As the binary system has base r = 2. So, the two types of complements for the binary system are 2's complement and 1's complement.

4.1.1 's complement

The 1's complement of a number is found by changing all 1's to 0's and all 0's to 1's. This is called as taking complement or 1's complement. Example of 1's Complement is as follows.

Example: Find the 1's complement of **$(10101)_2$**?

Answer:



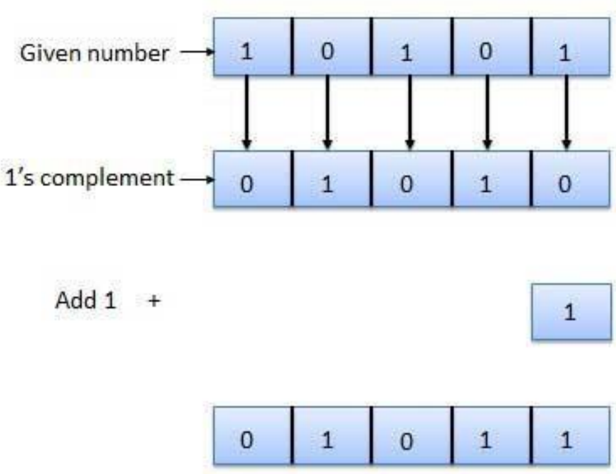
1's complement of **$(10101)_2$ =$(01010)_2$**

4.2.2 's complement

The 2's complement of binary number is obtained by adding 1 to the Least Significant Bit (LSB) of 1's complement of the number.

2's complement = 1's complement + 1

Example: Find the 1's complement of **$(10101)_2$**?



2's complement of $\mathbf{(10101)_2 = (01011)_2}$

5. Exercises

1- Convert $(1551)_{10}$ to:
   a. Binary number.
   b. Octal number.
   c. Hexadecimal number.
   d. BCD.

2- Convert $(110101.01)_2$ to:
   a. Decimal number.
   b. Octal number.
   c. Hexadecimal number.

3- What are the numbering systems that could represent the following numbers?
   a. (10110110).
   b. (1202).
   c. (7832).

4- Perform the following arithmetic operations:
   a. $(001101.110)_2 + (00010)_2$
   b. $(753)_8 - (632)_8$
   c. $(75B.03)_{16} - (632.A)_{16}$
   d. $(75.B3)_{16} + (63.2A)_{16}$

5- Find the result of adding (234) and (472) if you know that the two numbers are:
   a. Decimal.
   b. Octal.
   c. Hexadecimal.

6- Convert the following number to BCD:

a. $(101110)_2$.
b. $(354)_8$.
c. $(ABF)_{16}$.

7- Find the results of the following:
a. $(101.110)_2 \times (0.101)_2$.
b. $(101.110)_2 \div (0.101)_2$ allow 3 digits after decimal.
c. $(11.10)_2 \times (1.101)_2$.
d. $(11.11)_2 \div (1.01)_2$ allow 3 digits after decimal.

8- Find the 1's and 2's complement of the following:
a. $(100111010)_2$.
b. $(0011011)_2$.